NOTATION

t, time; M, point in space; $\rho(M, t)$, density of medium; $p(M, t)$, pressure; $\mathbf{V}(M, t)$, velocity of medium; $T(M, t)$, temperature of medium; $a(M, t) = \lambda/C_p\rho(M, t)$, thermal diffusivity of medium; λ, thermal conductivity of medium; C_p, specific heat; $\alpha(M, t)$, coefficient of absorption; σ_0, Stefan–Boltzmann constant; $A(M, t)$, absorptive capability of boundary surface; V, volume occupied by medium; μ, dynamic viscosity coefficient; q, efficiency of internal heat sources (sinks); Diss $f(\mathbf{V})$, dissipative Rayleigh function; $T_i(M)$, initial temperature; $T_W(M, t)$, temperature of boundary surface; $T_c(M, t)$, temperature of surrounding medium; α_{em}, heat emission coefficient; $z(t) = T(0, t)$, temperature at boundary of semiinfinite body; $\Gamma(z)$ Euler gamma function; $G_1(\alpha, \gamma, z)$, degenerate hypergeometric function of the second sort; Erfc (z), complementary minor of error function.

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MATHEMATICAL MODELING OF RADIANT HEAT-EXCHANGE PROCESSES IN METALLURGICAL THERMOTECHNOLOGY

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Various models of the heat-exchange process in metallurgical furnaces are considered: old methods of calculation, the zonal method, and mathematical models of radient and complex heat exchange in plane and cylindrical channels.

A new concept of the "mathematical model" has appeared in contemporary scientific literature. The creation of a mathematical model must be preceded by development of a physical picture of the phenomena (physical model) which determines the geometry of the system and peculiarities of the processes described by the mathematical model, let us say the character of the motion of the medium (stack gases), the values of physical constants, the rate of fuel burnup, etc. These processes are presented in a simplified manner, since an accurate description of the operation of the aggregate is impossible. This is the first stage of the problem. The second is the composition of equations describing the processes and boundary conditions. This is the mathematical modeling. The third stage is the solution of these equations, i.e., obtaining concrete results for model (furnace) operation. The three stages together comprise a method for calculating heat transfer in a furnace. Finally, the last step is adaptation of the model to the actual aggregate, i.e., verification of the calculation with experimental data with subsequent correction, bringing to life, as it were, the mathematical model developed.

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We usually represent the mathematical model as a set of complex equations with corresponding boundary conditions. Because of the introduction and development of the electronic computer, investigators now have the opportunity to use such models to produce concrete calculated results, although this possibility has only existed for the last 10-15 years. Before this, mathematical models were very simple. To calculate external heat exchange in flame furnaces, formulas of the type

$$Q_h = H\sigma_{app}(T_{ef}^4 - T_{ef}^4), \tag{1}$$

$$Q_h = H[\sigma_{rg}(T_g^4 - T_s^4) + \sigma_{rp}(T_p^4 - T_s^4)]. \tag{2}$$

were used. These formulas are still used today [1-3]. They rely on the assumption that the heat-exchange character with variable stack gas temperature is identical to the character of heat exchange with identical temperature over the volume. The second type of function used is

$$-WdT + q_{ch}dH = \sigma_{app}(T_g^4 - T_s^4)\,dH. \tag{3}$$

This formula determines heat exchange by a simplified one-dimensional scheme with no consideration of axial radiation fluxes.

The major peculiarity of heat-transfer calculations in metallurgical furnaces is the large role of internal heat exchange and the complex conditions determining the calculation of the latter. There is special difficulty in simultaneous solution of the problem of internal and external heat exchange.

A second peculiarity of developing mathematical heat-exchange models for furnaces is the multitude of furnace types: heat-treating furnaces with various numbers of zones, sectional furnaces, various types of thermal furnaces, open-hearth furnaces, etc.

From the viewpoint of heat-exchange calculation the following factors are important: 1) whether or not the process is stationary; 2) the form of the working space; 3) the dimensions and form of the bodies being heated; 4) the method of fuel supply and the burnup rate. These processes have not been studied sufficiently, so to describe the process of fuel burnup, most often empirical formulas are used, e.g., [4]:

for degree of burnup:

$$\bar{\chi} = 1 - \exp(-mx^2), \tag{4}$$

for coefficient of air flow rate

$$\bar{\alpha_g} = \alpha_0[1 - \exp(-mnx)], \tag{5}$$

where x is distance along the furnace length and m and n are coefficients.

The internal heat-exchange processes depend on the type of furnace. For heat treatment and thermal furnaces internal heat exchange is usually described by a Fourier equation with appropriately chosen boundary and initial conditions. Frequently this equation must be applied to billets of complex form. The problem is also complicated by the presence of scale which is usually formed in parallel with the metal heating process. The scale offers an extra thermal resistance and complicates the problem by the presence of the heat liberated during its formation. Available information on scale formation is still very insufficient. In calculating metal heating the temperature dependence of thermophysical properties is often considered. Due to the complexity of calculating metal heating, simplifications are made by use of functions based on the principles of a regular regime, e.g., [5]

$$E_r = (T_s - T_m)\left(\frac{\delta_{sc}}{\lambda_{sc}} + \psi\frac{R_m}{\lambda_m}\right)^{-1}. \tag{6}$$

This formula gives a simple relationship between the thermal load of the heating surface E_r and the temperatures T_s and T_m. It can be used with Fourier number values which are not too low.

The situation is even more complex in melting type furnaces, e.g., open hearths. Heating and melting of the metal takes place through a layer of scale of uncertain thickness and incompletely determined chemical composition. Chemical reactions with accompanying heat liberation and absorption take place throughout the thickness of the metal and scale.

In practical calculations, older methods of determining external heat exchange are often used, based on use of Eqs. (1) and (2). If we are calculating a one-chamber furnace, then the heat transfer equations are simultaneously solved with the balance equation. Here the choice of an effective radiation temperature remains unclear. Usually it is assumed that $T_{ef} = T_{ex}$, while sometimes $T_{ef} = \sqrt{T_{ex}T_t}$ is used. A more general form for T_{ef} was offered by Polyak and Shorin [6]:

$$T_{ef} = T_t^{(1-n)} T_{ex}^n \cdot \tag{7}$$

The value of the coefficient n depends on the hydrodynamics of the process in the furnace and the rapidity of fuel burnup. With intense mixing n = 1.0. In the case of multichamber methodical furnaces, Eqs. (1) and (2) must be applied to each chamber. Now the heat content of the gas at the chamber input will depend on the temperature of the exhaust gas from the preceding chamber, which compels simultaneous solution for all chambers. The problem is complicated by the fact that the internal heat exchange must be calculated simultaneously with the external.

At the All-Union Scientific-Research Institute of Metallurgical Thermotechnology external heat exchange is calculated with a one-dimensional scheme, with the furnace divided along its length into several calculation zones. In each zone an identical temperature is assumed for gas and metal, and the heat-exchange equations are written with consideration of heat transfer across the packing. Metal heating is calculated by solution of the Fourier equation. Axial heat flux from radiation and thermal conductivity are not considered. Methodical furnaces operate with a counterflow principle, which complicates the solution, since known metal and gas temperatures are found at opposite ends of the furnace. The problem is solved in two formulations: by specifying the metal surface temperature along the furnace length, or by specifying fuel flow rates in individual sections of the furnace. The calculation is performed for both the closest slab (layer) and for individual billets (furnaces with walking hearths and beams).

The development of zonal calculation methods began in the mid-1930s (Polyak, Timofeev). An especially complete solution of this problem can be found in the studies of Yu. A. Surinov. Initially, this method encompassed heat exchange between surfaces. To apply it to flame furnaces, it was necessary to solve the problem of heat exchange between volumes and consider convective heat transfer. The first application of the zonal method to flame furnaces was in the study of Hottel and Cohen in 1958 [7]. The calculation is based on solution of a system of nonlinear radiation equations, where the variables are the value of the intrinsic zone radiation and the resulting heat exchange. The coefficients used are generalized angular ones. In a number of studies [9-12] this method was used to consider the effect of various factors on heat exchange, and calculations by the zonal method were compared to other methods.

Another calculation method was proposed by Klekl' [13]. It is based on the equation

$$\sum_j A_{ji}T_j^4 - A_iT_i^4 + \sum_j g_{ji}T_j + Q_i = 0. \tag{8}$$

In [14, 15] selective radiation exchange coefficients A^{Σ} were introduced. These are substituted in Eq. (8) for the gray coefficients A. For volume zones

$$A_{ji}^{\Sigma} = 4\sigma_0 V_j \sum_z \overline{f}_{ji}^{\lambda} \overline{k}_j^{\lambda} \alpha_j^{\lambda},$$

and for surface zones

$$A_{ji}^{\Sigma} = \sigma_0 F_j \sum_z \overline{f}_{ji}^{\lambda} \overline{\varepsilon}_j^{\lambda} \alpha_j^{\lambda},$$

where $\overline{f}_{ij}^{\lambda}$ is the mean (over the limits of the radiation band) of the corrected resolving angular coefficient of radiation from zone j to zone i. This is the ratio of the amount of energy absorbed by zone i to the amount of energy radiated by zone j. In Klekl's scheme these coefficients are found by the Monte Carlo method. Later Lisienko and Zhuravlev [16] introduced changes in these coefficients. The Monte Carlo method was used to find generalized angular coefficients, and the resolving coefficients were found by solution of a system of equations.

In a zonal calculation with consideration of radiation selectivity, in contrast to Hottel's model of "three gray gases" the position of the gas radiation bands, the bandwidths, and the integral index of absorption are taken close to their real values. Using the integral absorption coefficients [17], the calculated absorption coefficient is determined over the effective bandwidth, and the mean emissivity $\bar{\varepsilon}^*_{\lambda z}$ over effective beam length is found. The values thus determined are corrected with experimental values of total integral emissivity [14, 15]

$$\bar{\varepsilon}_{\lambda z} = \bar{\varepsilon}^*_{\lambda z} \frac{\varepsilon_{exp}}{\varepsilon_{calc}}, \tag{9}$$

where ε_{calc} is the calculated integral emissivity found by summation of the $\bar{\varepsilon}^*_{\lambda z}$ values.

A method was also developed for approximate calculation of the gas selective properties using a simplified two-band model of the radiation spectrum. The fractions of black body radiation in the region of the gas absorption bands needed in such calculations were determined, whereupon formulas were obtained for the apparent radiant heat-exchange coefficients in a system of three bodies: gas–packing–metal [18].

The tendency toward consideration of arbitrary flame configuration and complex form of metal and packing led to the creation of the node method of calculation [19]. Surfaces in the radiating system are specified by equations to the second order, and in both the volume of the radiating medium and on each zone surface a certain number of node points are distinguished, relative to which the calculation of local temperature or thermal flux is performed. The number of nodes used is determined foremost by the form and position of the flame, the required degree of approximation of the temperature distribution and absorption coefficients in the volume, and by the nonuniformity of the temperature distribution over the surfaces. Local temperatures at the nodes are found by solution of a system of nonlinear heat transfer and thermal balance equations.

In studies performed by the Ivanov Energy Institute, a simplification was introduced, by replacing volume zones by surfaces with isotropic radiation.

In [20], for the first time in construction of a multizone model of a heat treatment furnace, the internal resistance of the massive metal was considered by using massive body form coefficients in Eq. (6). In such zonal furnace models it has been possible to consider flame length and flame radiation characteristics variable over length, the flame position relative to the surface being heated and the packing, the effect of convective heat exchange, and selectivity of radiation.

In [21] an approach was employed for solution of the problem of metal heating in a model of a methodic heat-treatment furnace which used the zonal method for calculation of external heat exchange and the Fourier equation for calculation of internal heat exchange. This was done by dividing the furnace length into n zones with identical gas temperature within each zone. The metal temperature over the length of each zone was assumed constant, but varying with height. Metal heating over a time τ_0/n (where τ_0 is the duration of the metal's stay in the furnace) was considered, after which heating of the metal mass continues in the neighboring zone, where the initial state is assumed identical to the final state in the preceding zone.

In [22] a solution of the external problem by the node method was linked to a solution of the internal problem by the grid method. The nonlinear boundary conditions during heating were written as local values, for the vicinity of a surface node N of a metal zone of arbitrary form, i.e., as nodal nonlinear boundary conditions. The method of zonal calculation of furnaces developed and perfected at the Ural Polytechnic Institute is used at present for construction of mathematical models and calculation-theoretical analysis of heat-exchange processes in various types of furnaces.

Recently, methods have been developed for exact calculation of complex heat exchange in plane channels with moving media. Such a channel can imitate the operation of certain furnace types quite well. The equation

$$\frac{\partial}{\partial y} \lambda \frac{\partial T}{\partial y} - w c \rho \frac{\partial T}{\partial z} - \operatorname{div} \mathbf{E} = 0 \tag{10}$$

describes the process of complex heat exchange in a plane slit channel. The medium moves along the z axis, and heat transfer by thermal conductivity in the y direction perpendicular to the direction of medium motion is considered. Initially, to determine div E the Schwarzschild–Schuster scheme was used without consideration of temperature change over channel length [23], while later temperature change along the channel was considered [24]. The dependence of the heat-exchange quantities on the defining criteria Bo, Bu, Pe was obtained and solutions were also achieved for a cylindrical channel [25].

NOTATION

Q_h, Q_{ex}, amount of heat received by surface to be heated and exterior; T_g, T_m, T_p, T_s, T_{ex}, T_t, absolute temperatures of gas, metal, packing, surface, exhaust gas, and theoretical value; σ_{app}, apparent radiant heat-exchange coefficient; σ_{rg}, σ_{rp}, apparent radiant heat-exchange coefficients between gas and metal and between packing and metal; q_{ch}, chemical heat liberation per m² of surface heated; E_r, density of resultant heat exchange; **E**, radiant flux vector; α_i^λ, fraction of blackbody radiation in region of band z at temperature of given radiating zone; $\overline{k}_i^\lambda$ and $\overline{\varepsilon}_i^\lambda$, mean (over band) absorption coefficient and emissivity; α_0, initial air flow rate coefficient; δ_{sc} and λ_{sc}, thickness and thermal conductivity of scale layer; λ_m, thermal conductivity of metal; R_m, characteristic dimension of billet form; ψ, form factor; F_i and V_i, surface and volume; W, water equivalent of flow; w, c, ρ, velocity, heat capacity, and density of medium; H, heating surface.

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MATHEMATICAL MODELING OF RADIANT HEAT EXCHANGE IN THERMOTECHNICAL EQUIPMENT

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An investigation is performed and solution presented of the most general formulation of the problem of radiant heat exchange in a chamber of rectangular cross section filled with an attenuating medium.

Introduction. The present study is an application of the third form of Surinov's generalized zonal method [3-5] to numerical study and solution of the eighth formulation of the problem of [1, 2] of radiant heat exchange in a chamber of rectangular cross section in the shape of a rectangular parallelepiped, filled with an inhomogeneous absorbing and scattering gray medium, the volume V of which is divided along the chamber height into three volume zones V_j ($j = 15, 16, 17$; $V = \sum_{j=15}^{17} V_j$).

The lateral surface of the chamber is divided by the volume zones into three portions each of which consists of four boundary zones. Thus, the boundary surface of the chamber F is divided into 14 zones, two of which, F_{13}, F_{14}, are the chamber bases, while 12 represent the lateral surface. The emissivity A_i ($i = 1, 2, \ldots, 14$) for all these zones, considered as optically homogeneous, diffusely radiating and reflecting gray bodies, is specified.

Formulation of Problem. The eighth formulation of the problem considered below is characterized by a mixed specification of both boundary and internal (volume) optical and energy characteristics of the radiation field. It is required to determine the temperature field for those boundary and volume zones for which the resultant radiant flux has been specified, and to determine the resultant fluxes (and, correspondingly, the densities of the resultant and other forms of hemispherical and volume radiation) for those boundary and volume zones for which the temperature was initially specified.

It will be assumed that for the surfaces F_2, F_6, F_{10}, F_4, F_8, F_{12}, F_{13}, and F_{14} considered as isothermal, the temperatures are specified, while for the remaining boundary surfaces the resultant radiant fluxes are specified and the zones F_1, F_5, and F_9 are considered adiabatic.

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